Numerical Investigation of Spheromak Formation Efficiency

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UMBC HPCF Research Forum
May 1, 2015

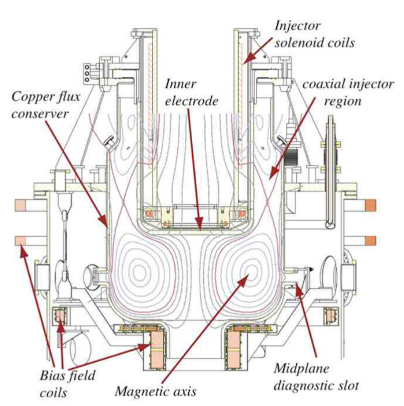
Spheromaks provide an opportunity to explore fusion relevant physics and technology in a compact device.

- A spheromak plasma **forms through self-organization** and magnetic reconnection.
- Spheromaks achieve high plasma β through:
  - Tailoring the current profile (development of feedback and control technology)
  - A kinetic ion population (energetic α particles in fusion plasmas)
- High-β operation makes the spheromak a good candidate for a FNSF for reactor component development and testing.
- The lack of toroidal field coils and a central solenoid means that the vacuum vessel is simply connected, which translates to a cheaper, more compact device with greater engineering simplicity (e.g. blanket design) and is easier to maintain.[1]

[1] S. Woodruff et. al. *J. Fus. En.* **29**. 2010.

This study seeks to explore spheromak formation through coaxial helicity injection.

- Two electrodes connected by vacuum magnetic flux are biased relative to each other.
- Current flows along the magnetic field lines, producing an expanding flux bubble.
- Identical to CHI in tokamaks, except there is no vacuum toroidal field.
- The injected current must be reduced after the formation 'pulse' in order to allow good confinement, i.e. a large region of closed flux surfaces.



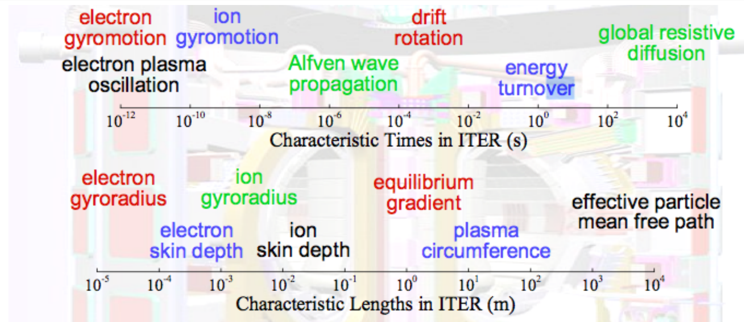
Sustained Spheromak Physics eXperiment (SSPX) Design[2]

[2] E.B. Hoooper et. al. *PPCF*. **54**. 2012.

The goal of this research is to optimize spheromak performance and improve confinement.

- The rate of helicity injection will be maximized by through control of the CHI gun current parameters and magnetic flux.
- We will expand beyond the achievable operational regimes of previous experiments in order to find candidate modes of operation for **future experimental studies**.
- In addition to the capacitor bank model, we also consider idealized injector models, i.e. I_g is prescribed and there is no feedback from the plasma.
- While this study initially considers the SSPX flux conserver geometry, it is **not a direct continuation** of previous SSPX campaigns and other designs will be examined.

For a fusion plasma, the physics of interest covers many orders of magnitude in spatial and temporal scales.



(image courtesy C.R. Sovinec. UW Plasma Seminar. 2007.)

- The PDE system is characterized by extreme stiffness and anisotropy.
- Some phenomena, e.g. magnetic reconnection, can occur at the smallest spatial scales, but influence global mode behavior.

The computations solve the low-frequency MHD and two-fluid models starting from vacuum magnetic field and 'cold fluid.'

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = D_n \nabla^2 n$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \mathbf{J}\times\mathbf{B} - \nabla p + \nabla\cdot\underline{\mathbf{\Pi}}(\underline{\mathbf{W}}) \qquad \text{where} \quad \underline{\mathbf{W}} = \nabla\mathbf{v} + \nabla\mathbf{v}^T - (2/3)(\nabla\cdot\mathbf{v})\underline{\mathbf{I}}$$

$$\frac{2n}{3}\left(\frac{\partial T_e}{\partial t} + \mathbf{v}_e\cdot\nabla T_e\right) = -nT_e\nabla\cdot\mathbf{v}_e - \nabla\cdot\left[\kappa_{\parallel e}\hat{\mathbf{b}}\hat{\mathbf{b}} + \kappa_{\perp e}\underline{\mathbf{I}}\right]\cdot\nabla T_e + n\sigma(T_i - T_e) + \eta J^2$$

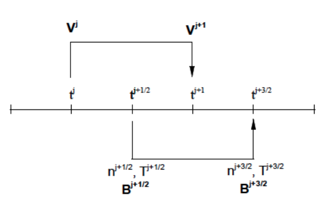
$$\frac{2n}{3}\left(\frac{\partial T_i}{\partial t} + \mathbf{v}_i\cdot\nabla T_i\right) = -nT_i\nabla\cdot\mathbf{v}_i - \nabla\cdot\left[\kappa_{\parallel i}\hat{\mathbf{b}}\hat{\mathbf{b}} + \kappa_{\perp i}\underline{\mathbf{I}}\right]\cdot\nabla T_i + n\sigma(T_e - T_i)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\left[\eta\mathbf{J} - \mathbf{v}\times\mathbf{B} + \frac{\mathbf{J}\times\mathbf{B} - \nabla p_e}{n_e e} + \frac{m_e}{n_e e^2}\frac{\partial \mathbf{J}}{\partial t}\right] = \kappa_{\nabla\cdot\mathbf{B}}\nabla^2\nabla\cdot\mathbf{B} \qquad \text{where} \quad \mathbf{J} = \mu_0^{-1}\nabla\times\mathbf{B}$$

- The computations use realistic, evolving, locally-computed transport coefficients.
- Neutrals, ionization, and recombination are not modeled.
- The NIMROD code (nimrodteam.org) is used to solve these systems.

The Non-Ideal Magnetohydrodynamic with Rotation, Open Discussion (NIMROD) code simulates macroscopic plasma dynamics with an extended MHD model.

- NIMROD uses 2D spectral finite elements in the poloidal plane with a finite Fourier series in the periodic dimension.[3]
- This representation allows high order convergence even with non-uniform, curved isoparametric meshes.



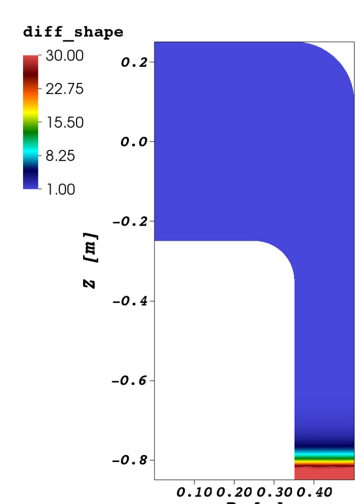
Implicit Leapfrog Algorithm

- This representation also accurately reproduces anisotropic transport without requiring alignment between the mesh and magnetic field.
- The code uses an implicit leapfrog algorithm that is linearly stable for arbitrarily large time-steps and free of numerical dissipation when the advection and magnetic diffusion terms are time-centered.[4]

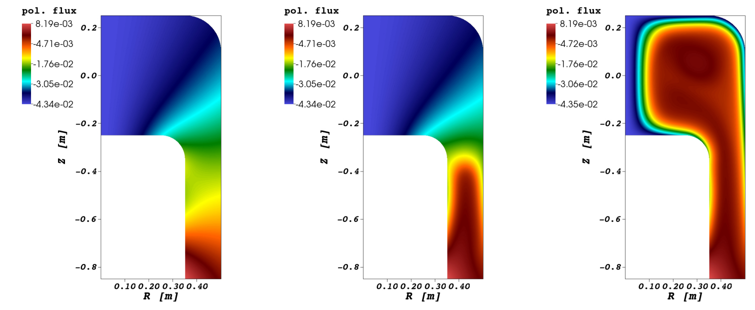
[3] Sovinec et. al. *J. Comp. Physics*. 2004.
[4] Sovinec and King. *J. Comp. Physics*. 2010.

Only the injector is prescribed: all dynamics follow self-consistently from the model.

- The injector is simulated by specifying $RB_\phi = \mu_0 I_g / 2\pi$ along the injector boundary.
- To create an insulating gap to encourage the expansion of the flux bubble into the domain, resistivity is enhanced along the injector boundary: $\eta \rightarrow \eta + (D_s - 1)\,\eta_{inj}$.
- The density boundary condition along the injector edge is initially no-flux, but transitions to Dirichlet when $n < n_{crit}$.



The timescale in the computations is in better agreement with the experimental plasma evolution than previous campaigns.

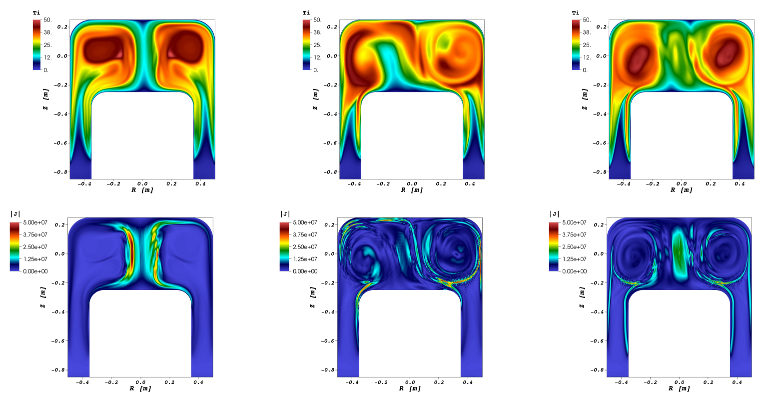


$t = 0\ \mu s$ $t = 52\ \mu s$ $t = 520\ \mu s$

- Typically $\sim 150\ \mu s$ for ejection of the flux bubble from the injector.[5]
- Typically $\sim 300\ \mu s$ for the edge magnetic field to peak and onset of the column mode instability.

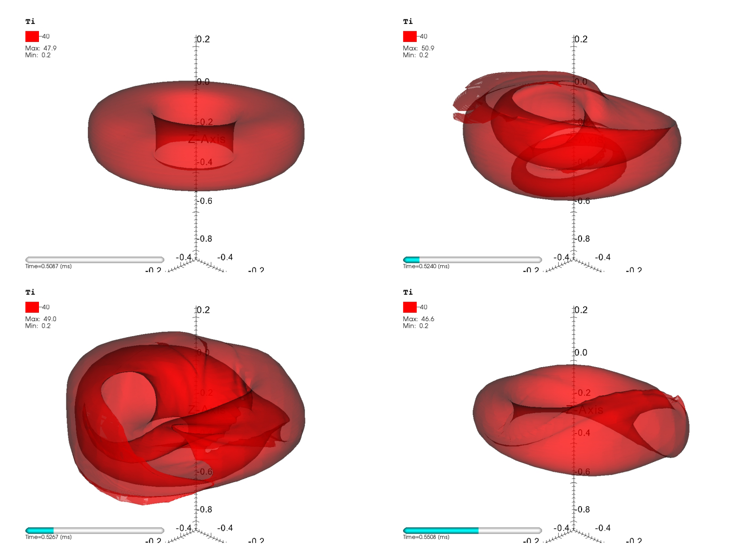
[5] R.D. Wood et. al. *Nucl. Fus.* 2005.

The plasmas undergoes a dramatic change in magnetic topology during the column mode, an $n_\phi = 1$ instability.

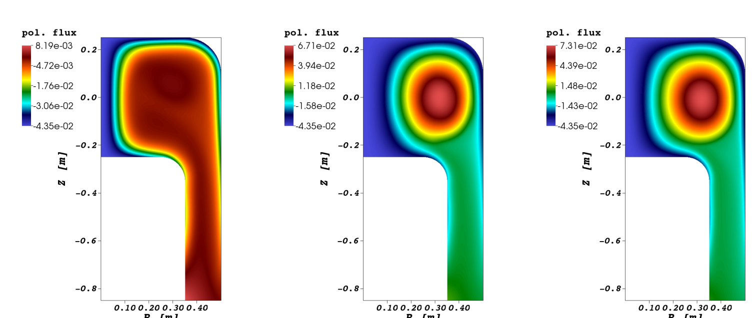


$t = 520\ \mu s$ $t = 528\ \mu s$ $t = 560\ \mu s$

After the column mode, the plasma settles into a helical equilibrium state.



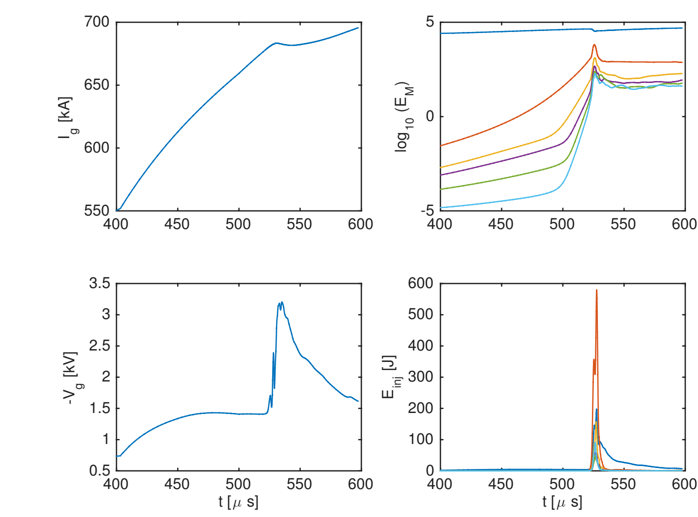
The column mode produces significant magnetic flux amplification.



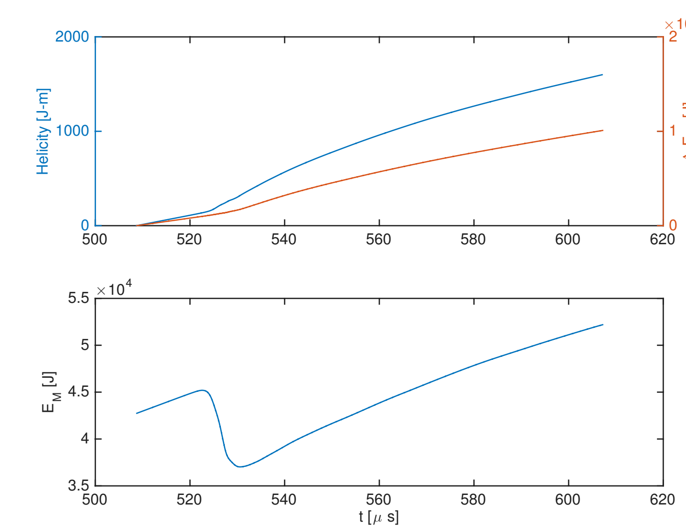
$t = 520\ \mu s$ $t = 528\ \mu s$ $t = 560\ \mu s$

- Flux amplification occurs over a very short timescale ($\sim 10\ \mu s$).
- The amplified flux decays very slowly relative to the relaxation dynamics.

The column mode occurs very quickly ($\sim$ 10s μs) and produces a large injector voltage spike.

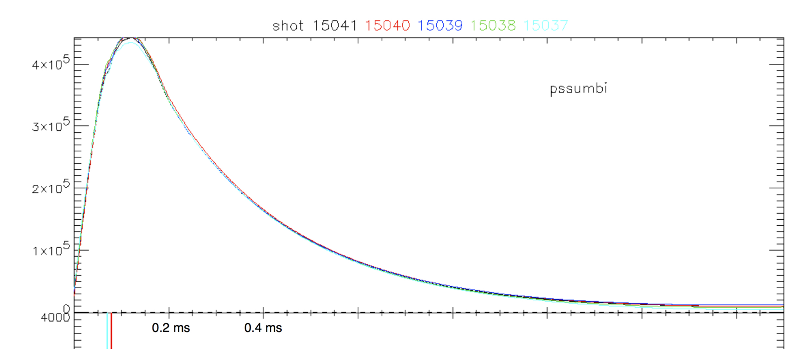


While magnetic energy is rapidly depleted during the column mode, the evolution of magnetic helicity is dominated by its injection.

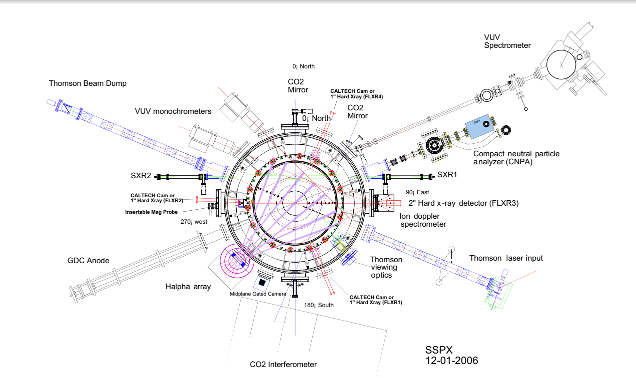


We are in the process of validating our numerical model against SSPX experimental data.

- Courtesy of LLNL, we have access to the entire SSPX experimental database and intend to model a wide variety of shots.
- This validation step is necessary to have confidence when modeling new flux conserver geometries and/or operational regimes.
- For single-pulse formation shots in SSPX, the plasma current is well approximated by a modified Boltzmann function.



To substantiate our numerical mode, we need to make direct comparisons to experimental data.

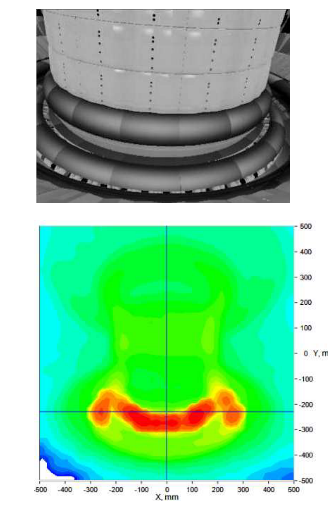


Top-Down View of Selected SSPX Diagnostics

- Many of these diagnostics measure line or field-of-view integrated emissions that in some cases depend on multiple fields.
- Therefore, it is necessary to reduce the vast set of computational data in order to make meaningful comparisons.

Our collaborators at Woodruff Scientific, Inc. have been developing a wide array of synthetic diagnostics.

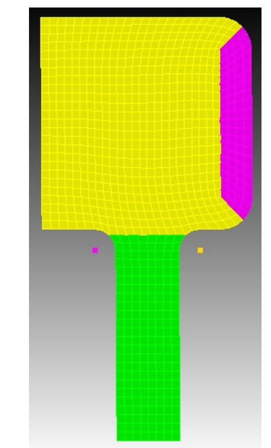
- The list of synthetic diagnostics includes, non-exclusively:
  - Interferometer
  - Polarimeter
  - Thompson scattering
  - Bolometer
  - VIS/IR Camera
- It may be necessary to temporally average over several measurements to reproduce expose time effects.



FOV showing Reflected Light

Though we are validating our numerical model with the SSPX flux conserver geometry, the goal is to explore different geometries.

- For example, moving the injector closer to the geometric axis and narrowing its 'throat' is expected to improve discharge performance.
- Changing the direction of expansion away from the geometric axis and increasing its expansion ratio (i.e. the relative amount the flux bubble must expand to fill the flux conserver) will decrease the speed at which the plasma compresses at the geometric axis.
- This should allow more helicity and magnetic flux to be injected before the onset of the column mode.



An SSPX-like flux conserver with $R_{inj} = 0.25$ m

Summary & Future Work

- The computations reproduce the column mode in previous SSPX simulations by EBH & BIC, but with timescales in better agreement to experimental observations.
- Development to the code allows for a much higher $\partial I_g/\partial t$ in simulations, which will aid in developing and exploring an operating regime space for the injector parameters.
- We are in the process of validating of numerical model against a variety of SSPX discharges.
- Our validation efforts will utilize the synthetic diagnostics developed by WSI for direct comparison to the experiment.
- (Very crudely) Explore and optimize the injector geometry, e.g. an oblate flux conserver.
- Explore additional physical effects (e.g. full Braginskii $\mathbf{\Pi}$, kinetic effects)

Acknowledgements

This work is supported under the Defense Advanced Research Projects Agency (DARPA) under Grant Number N66001-14-1-4044.

The hardware used in the computational studies is part of the UMBC High Performance Computing Facility (HPCF). The facility is supported by the US National Science Foundation through the MRI program (grant nos. CNS-0821258 and CNS-1228778) and the SCREMS program (grant no. DMS-0821311), with additional substantial support from the University of Maryland, Baltimore County (UMBC). See www.umbc.edu/hpcf for more information on HPCF and the projects using its resources.

Computational resources have also been provided by the National Energy Research Scientific Computing Center (NERCS), which is supported by the Office of Science of the US Department of Energy under Contract No DE-AC02-05CH11231.

Access to Sustained Spheromak Physics eXperiment (SSPX) data is provided through the Lawrence Livermore National Laboratory (LLNL) Visiting Scientist Program under Collaboration Request Number 5920700.