

Enhanced Data Exploration and Visualization Tool for Large Spatio-Temporal Climate Data

REU Site: Interdisciplinary Program in High Performance Computing

Ethan Crasto¹, Sydney Kahmann², Paula Rodriguez³, Benjamin Smith⁴,

Graduate assistants: Sai K. Popuri⁵, Nadeesri Wijekoon⁵,

Faculty mentor: Dr. Nagaraj K. Neerchal⁵

Client: Amita Mehta⁶

¹Department of Mathematics and Statistics, The College of New Jersey

²Department of Statistics, University of California, Los Angeles,

³Department of Mathematics and Department of Computer Science,
California State University Channel Islands

⁴Department of Mathematics, Towson University,

⁵Department of Mathematics and Statistics, University of Maryland, Baltimore County,

⁶Joint Center for Earth Systems Technology (JCET)

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Abstract

Predictions of climate variables like precipitation and maximum/minimum temperatures play crucial role in assessing the impact of decadal climate changes on regional water availability. This technical report describes a Graphical User Interface (GUI) called CMIViz developed as part of the 2016 REU program at UMBC. CMIViz is an R tool used for exploration and visualization of spatio-temporal climate data from the Missouri River Basin (MRB). The tool is developed using the R package ‘Shiny’, which facilitates access on a web browser. Since prediction of precipitation is more challenging than the prediction of maximum/minimum temperatures, CMIViz provides more visualization options for precipitation. Specifically, the tool provides an easy inter-comparison of data from the Global Climate Models (GCM): MIROC5, HadCM3, and NCAR-CCSM4 in terms of bias relative to the observed data, root mean-squared error (RMSE), and other measures of interest for daily precipitation. The tool has options to explore the temporal trends and autocorrelation patterns given a location and spatial patterns using contour plots, surface plots, and semivariograms given a time point. CMIViz also provides visualization of canonical correlation analysis (CCA) to help find similarities between the models.

Keywords: Graphical user interface (GUI), Global Climate Models (GCM), Missouri River Basin (MRB), spatio-temporal analysis, exploratory data analysis (EDA), MIROC5, HadCM3, NCAR-CCSM4.

1 Introduction

The Missouri River Basin (MRB) is a vast region spreading across ten U.S. states and accounting for 28% of the nation's farmland [14]. The region, shown in Figure 1.1¹ is responsible for producing approximately 46% of wheat, 22% of corn, and 34% of cattle nationwide. Since much of the region is not irrigated, there is significant reliance on rainfall. This necessitates a need to assess the availability of water. For example, during periods of drought, soil is more susceptible to wind erosion, which makes the soil lose vital minerals, thereby hurting the fertility of the region [14]. Therefore, to protect and maintain current crop yields there is a need to better understand the impact of climate changes on the water availability in the region [11].

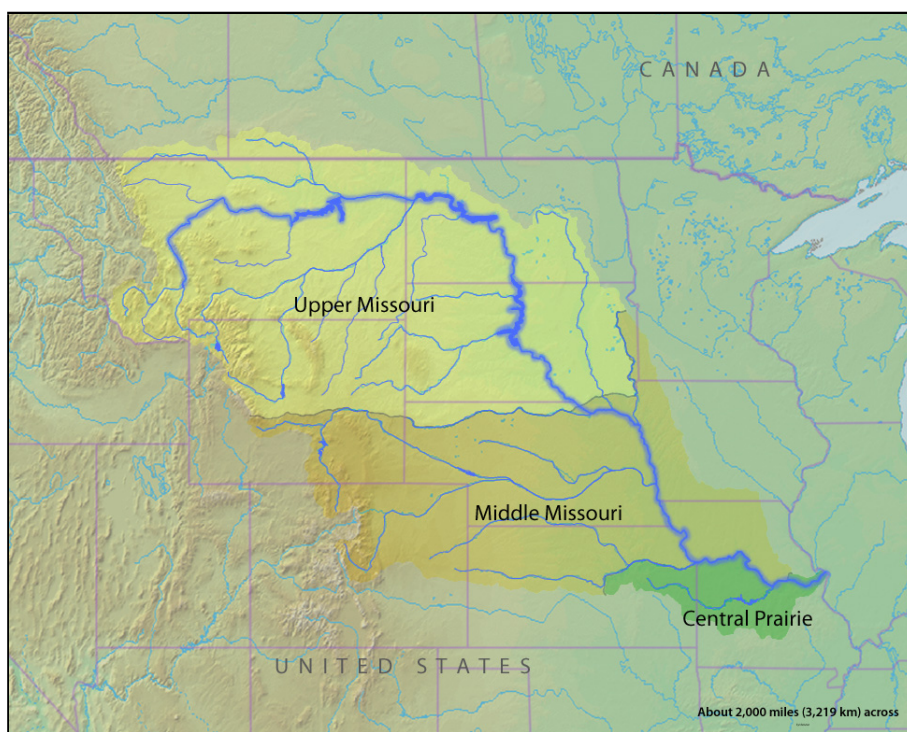


Figure 1.1: The Missouri River Basin is a prominent agricultural region of the United States.

Prediction of maximum/minimum temperatures and precipitation in the MRB was studied by the 2014 UMBC REU [4] and 2015 UMBC REU [3] teams using simulated data from two climate models: Hadley Center Coupled Model (HadCM3; [7]) and the Model of Interdisciplinary Research on Climate (MIROC5; [17]). They both noted that while predictions for maximum/minimum temperatures were found to be satisfactory, precipitation was challenging to predict. Recognizing the need to aid the modeling efforts to predict precipitation with visualization options to explore various spatial and temporal relationships between the observed and the data provided by GCMs, CMIViz extends the basic GUI tool developed

¹Image from https://en.wikipedia.org/wiki/Missouri_River

by the 2014 UMBC REU team [4], which was primarily used to help prepare data for model fitting as opposed to data visualization. We have used the R package ‘Shiny’ [1] to make CMIViz web-based so that it can be hosted on a website. The tool provides several options to perform exploratory analysis of spatio-temporal observed and data provided by GCMs. Specifically, the tool focuses on: 1. inter-comparison of GCMs: MIROC5, HadCM3, and NCAR-CCSM4 [5] and with the observed data and 2. visualization options to help identify the underlying spatio-temporal correlations and patterns.

The rest of the report is organized as follows: Section 2 explains the premise and goals of the project. Section 3 discusses the data source and the GUI implementation in ‘Shiny’. Section 4 outlines the functionality provided in CMIViz. Section 5 describes possible future directions for CMIViz.

2 Background

A multi-institute team consisting of the Center for Research on the Changing Earth System (CRCES), Texas A&M University, UMBC-JCET, and the National Drought Mitigation Center (NDMC), supported by the US Department of Agriculture-National Institute for Food and Agriculture (USDA-NIFA), have been working on assessing the impacts of decadal climate variability on water availability and crop yields in the Missouri River Basin [11]. The team from UMBC-JCET have used daily and monthly low resolution data on several climate variables provided by Global Climate Models (GCM) to build prediction models for precipitation and maximum/minimum temperatures, which are used as inputs to the Soil and Water Assessment Tool (SWAT; [6]) to perform hydrological assessment studies in MRB. Global Climate Models are deterministic mathematical models based on physical laws of nature and provide simulations of future global climate patterns. Simulated data from GCMs are often used by statistical methods to forecast climate variables.

Previously, the UMBC REU teams from 2014 [4] and 2015 [3] have developed prediction methods for precipitation, maximum, and minimum temperatures over MRB using simulated data from HadCM3 and MIROC5 at the daily and monthly levels, respectively. The REU team from 2014 also developed a GUI to preprocess the data with some basic visualization features, which the REU team from 2015 modified for monthly level prediction. Both the teams noted that while predictions of maximum/minimum temperatures were found to be satisfactory, predictions of precipitation were not sufficiently accurate, possibly because of its semi-continuous (point mass at 0) property. Also, the prediction methods for precipitation used in [4] and [3] are fitted at each location independently and do not explicitly model the spatial and temporal dependence. A major component of CMIViz is the suite of visualization options and exploratory data analysis (EDA) for the precipitation data to help explore its spatial and temporal patterns. As a result, we hope CMIViz would complement the advanced prediction modeling currently being pursued for precipitation.

EDA is a means of investigating the data, often visually, before performing inferential statistics or model building [15]. EDA can be used to check key assumptions in traditional statistical analysis or discover hidden patterns in the data. For instance, in fields like social

sciences, EDA is often used to determine if the distribution of a parameter of interest is normal [16]. Unfortunately, EDA is often overlooked before conducting statistical analyses, resulting in problematic models and conclusions [8]. EDA is particularly relevant for spatio-temporal data because neighboring observations tend to share certain characteristics [2]. For example, correlations in space and time violate the traditional assumptions of independent and identically distributed errors. As a result, people seeking to model spatio-temporal data could benefit from using EDA to detect correlations within the data before using other forms of analysis to better ensure the validity of the model. To this end, one of our goals has been to provide visualization options specifically to explore the semi-continuous nature of the precipitation data. To better facilitate the inter-comparison of GCMs, we have added NCAR-CCSM4 to CMIViz in addition to HadCM3 and MIROC5. In order to increase the reach of CMIViz, we have developed it using the R package ‘Shiny’, which enables the tool to be accessed on a web browser. For example, it is possible to host CMIViz on the cluster maya at UMBC and make it accessible on an intranet setup to a restricted audience.

3 Implementation Methodology

3.1 Data

Currently, CMIViz provides visualization of the monthly level data from the GCMs: MIROC5, HadCM3, and NCAR-CCSM4. The temporal coverage of the data is 1950-2005 and the spatial resolution is $0.125^\circ(\text{longitude}) \times 0.125^\circ(\text{latitude})$, making it $12\text{km} \times 12\text{km}$ gridded data. The geographical region covered is restricted to MRB. Table 3.1 shows the longitude and latitude bounds of the region for each GCM. Table 3.2 shows the native spatial resolutions of the GCMs, which are spatially interpolated to the regional resolution of $0.125^\circ(\text{longitude}) \times 0.125^\circ(\text{latitude})$. The original data for the three GCMs is sourced from the Coupled Model Intercomparison Project Phase 5 (CMIP5; [13]). The observed precipitation data are provided by [10]. All the datasets are processed and converted as NetCDF files and are made accessible to CMIViz at the initialization stage. The climate variables that are available to analyze on CMIViz are maximum and minimum temperature measured in Kelvin, and precipitation measured in *mm/day*. Each GCM typically has several runs of simulations (predictions) of various climate variables for large time periods. Each such simulation run is called an ensemble. Our GUI uses an average of all the runs, called an ‘ensemble average’, as a consensus prediction of the corresponding GCM.

Model	Minimum Latitude	Maximum Latitude	Minimum Longitude	Maximum Longitude
MIROC5	34.9	50.0	-120	-83.8
HadCM3	32.5	52.5	-124	-82.5
NCAR-CCSM4	34.4	50.4	-121	-83.8

Table 3.1: Latitude and longitude ranges for MIROC5, HadCM3, and NCAR-CCSM4 GCMs.

	MIROC5	HadCM3	NCAR-CCSM4
Number of Latitudes	13	12	31
Latitude Increments	1.4	2.5	.942
Number of Longitudes	27	9	9
Longitude Increments	1.4	3.75	1.25

Table 3.2: Spatial resolutions of MIROC5, HadCM3, and NCAR-CCSM4.

3.2 Visualization Metrics

CMIViz provides several metrics to facilitate inter-comparison of the daily and monthly level precipitation, maximum/minimum temperatures across the three GCMs and the observed data. In this section, we describe the metrics relevant to the daily level precipitation data. Let $Y_m(s, t)$ be the observed precipitation, where m is the month, s is the location in the MRB, and t is the day in the month m . Let $X_m^{(p)}(s, t)$ be the simulated precipitation from the chosen GCM (MIROC5, HadCM3, NCAR-CCSM4), where $p \in \{1, 2, 3\}$ indicates the GCM. Let N be the number of days in the time period of study. Then the number of ‘wet’ days in the selected time period observed at the location s is given by $N_w = \sum_{t=1}^N I(Y_m(s, t) > 0)$, where I is the indicator function equal to 1 when $Y_m(s, t) > 0$ and 0 otherwise and the number of ‘dry’ days in the selected period is $N_d = 1 - N_w$.

The Root mean squared error (RMSE) is a means to quantify the error in the data provided by the GCM. RMSE is a function of the squares of the differences between the GCM and the observed values. RMSE of the simulated precipitation at the location s , from the p^{th} GCM, for the month m is calculated as

$$RMSE_m^{(p)}(s) = \sqrt{\frac{1}{N} \sum_{t=1}^N (X_m^{(p)}(s, t) - Y_m(s, t))^2} \quad (3.1)$$

Another useful measure to quantify the error in the simulated GCM data is ‘bias’, which is defined as the difference between the GCM and the observed values. Bias of the simulated precipitation at the location s , from the p^{th} GCM, for the month m is calculated as

$$BIAS_m^{(p)}(s) = \frac{1}{N} \sum_{t=1}^N (X_m^{(p)}(s, t) - Y_m(s, t)) \quad (3.2)$$

The proportion of dry days (no rain) observed at the location s in the month m is calculated as

$$DryProp_m(s) = \frac{1}{N} \sum_{t=1}^N I(Y_m(s, t) = 0) \quad (3.3)$$

The average intensity of the rain, measured for the days it rains at the location s in the month m is calculated as

$$Intensity_m(s) = \frac{\sum_{t=1}^N Y_m(s, t) I(Y_m(s, t) > 0)}{N_w} \quad (3.4)$$

An important feature of the simulated daily precipitation data from the GCMs is that the data is always strictly positive, that is, there are no dry days in the GCM data. The GCM's dry intensity measures the average GCM rainfall for the days when no rain was observed. At the location s and month m , the dry intensity is calculated as

$$ModelDryInt_m(s) = \frac{\sum_{t=1}^N X_m^{(p)}(s, t) I(Y_m(s, t) = 0)}{N_d} \quad (3.5)$$

Similarly, the GCM's wet intensity at the location s and month m measures the average GCM rainfall for the days it rained and is calculated as model wet intensity as follows:

$$ModelWetInt_m(s) = \frac{\sum_{t=1}^N X_m^{(p)}(s, t) I(Y_m(s, t) > 0)}{N_w} \quad (3.6)$$

Following are other visualization metrics available for the inter-comparison of GCMs on CMIViz: correlation between the GCM and the observed data and standard deviations of the bias and the GCM's dry intensity measures.

3.3 Implementation and Configuration

CMIViz is implemented in R 3.2.2 software using the package 'Shiny'. The GUI can be seen as an implementation of the Model-View-Controller (MVC; [9]) architecture. The package Shiny facilitates the MVC implementation by providing mechanisms to separate the graphical presentation to the user and the computational component. The implementation is based on the communication and interplay between two Shiny files: `ui.R` and `server.R`. The 'user-interface' `ui.R` code determines the interface design and defines the user inputs for the tool. Every input use case is defined and a suitable mode of selection is configured in `ui.R`. Every output component (e.g.: plot) is configured to be generated by an appropriate computational component in `server.R`. In other words, `server.R` contains code to define, generate, and save each rendered image. The data necessary is subsetted based on the user inputs and then is rendered as a plot using the code in the server file. See [1] for more details on the GUI development in R using the 'Shiny' package.

CMIViz is currently installed on the cluster maya at UMBC. In order to login to maya and access CMIViz, the user must have an account through the [UMBC High Performance Computing Facility](#). Since a web browser is not available on the cluster maya, users must set up a 'tunneling' mechanism by changing the settings in their browser and SSH connection to maya. For example, if an SSH tool like 'Putty' is used, the user should add a forwarding port (e.g.: 3128) to 'tunnel' the web traffic through maya at the specified port. Next, web browsers on the users' personal computer must be configured to route the web traffic via the SSH connection with maya. For example, in Firefox, the user must navigate to options→Advanced→Network→Settings. On this page the user must select 'Manual proxy configurations', enter 'localhost' for the SOCKS host, port number 3128 for 'SOCKS v5' and save the settings. Under these settings, web activity will be tunneled through maya, so to revert back to regular internet settings, the connection settings should be set back to 'no

proxy’. Once the SSH and browser settings are modified and the user is logged into maya, CMIViz is ready to be launched. On maya, the user must first navigate to the directory where the R source code for CMIViz is saved and run the command: `source('runGUI.R')` in an interactive R session. If successful, this command should show a message similar to ‘Listening on’ followed by an IP address. CMIViz can then be accessed by entering this IP address into the SOCKS proxy-enabled web browser on the user’s personal computer. Figure 3.1 shows the welcome page of CMIViz.

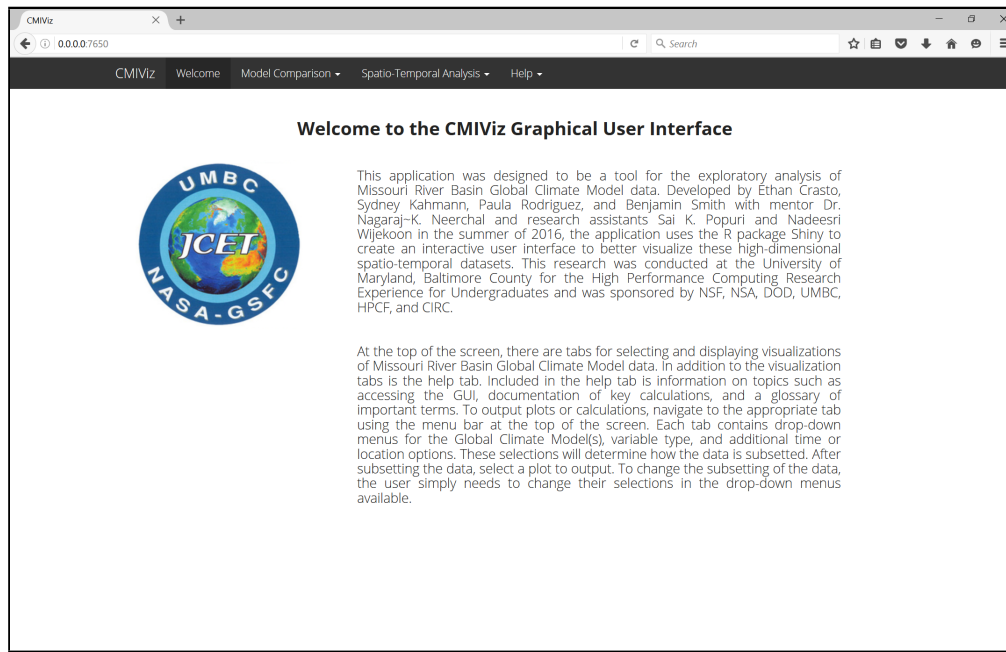


Figure 3.1: Home screen of CMIViz graphical user interface.

4 CMIViz

The functionalities provided by CMIViz can be broken down into the following categories:

- Inter-comparison of GCMs
- Temporal visualization
- Spatial analysis visualization
- Spatio-temporal visualization

Figure 4.1 shows a screenshot of CMIViz with all the available visualization options expanded on the menu bar. Appendix A has a listing of all types of visualization features offered by CMIViz.

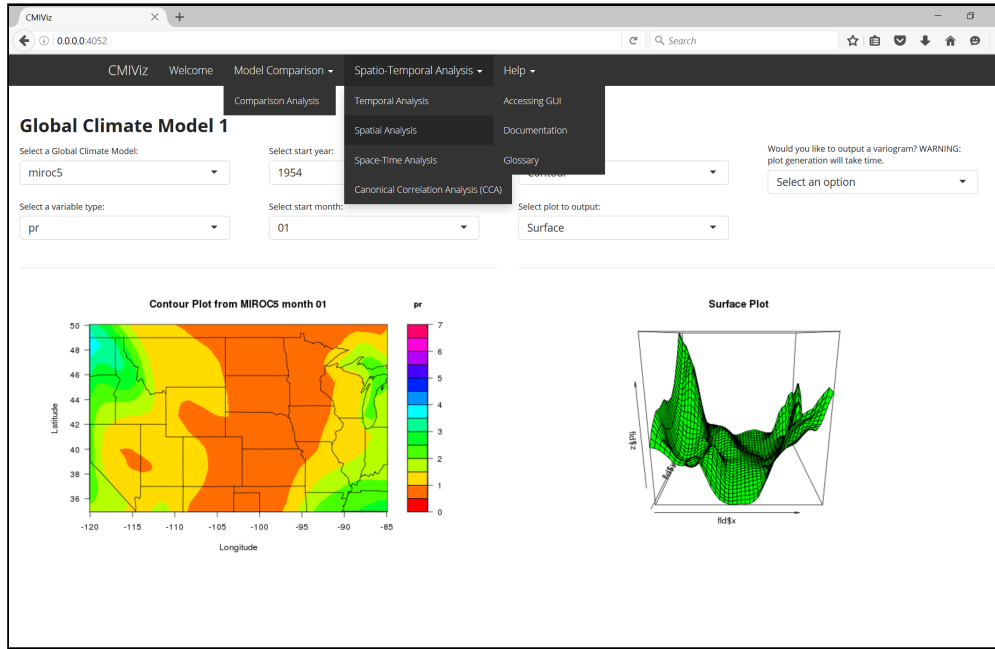


Figure 4.1: Full range of options for each dropdown menu on CMIViz.

4.1 Inter-comparison of GCM Data

Inter-comparison of the GCM data is available under ‘Model Comparison→Comparison Analysis’, shown in Figure 4.2, menu option on CMIViz. The user can compare the simulated data from MIROC5, HadCM3, and NCAR-CCSM4 in terms of their accuracy compared to the observed data. Statistics for model comparison are calculated and then plotted for each location over a spatial map. These statistical metrics for visualization are described in Section 3.2. All the features included in the Model Comparison tab allow the user to select a model, variable type, and time period.

4.2 Temporal Analysis

CMIViz provides options to perform basic exploratory and visualization analysis of time series data of a climate variable at a chosen location (latitude, longitude). These visualization options are available under ‘Spatio-Temporal Analysis→ Temporal Analysis’, shown in Figure 4.3, menu option on CMIViz. The user must select a climate variable, location, a time period, the GCM models to compare and the type of the visualization. Currently, the following options are available: time series plots showing the data averaged in time by month, histograms of the data, periodograms used to identify the dominant frequencies in the time series, and autocorrelation plots. If two models are selected, the data from both will be shown on the same plot. See [12] for more details on these time series visualization objects.

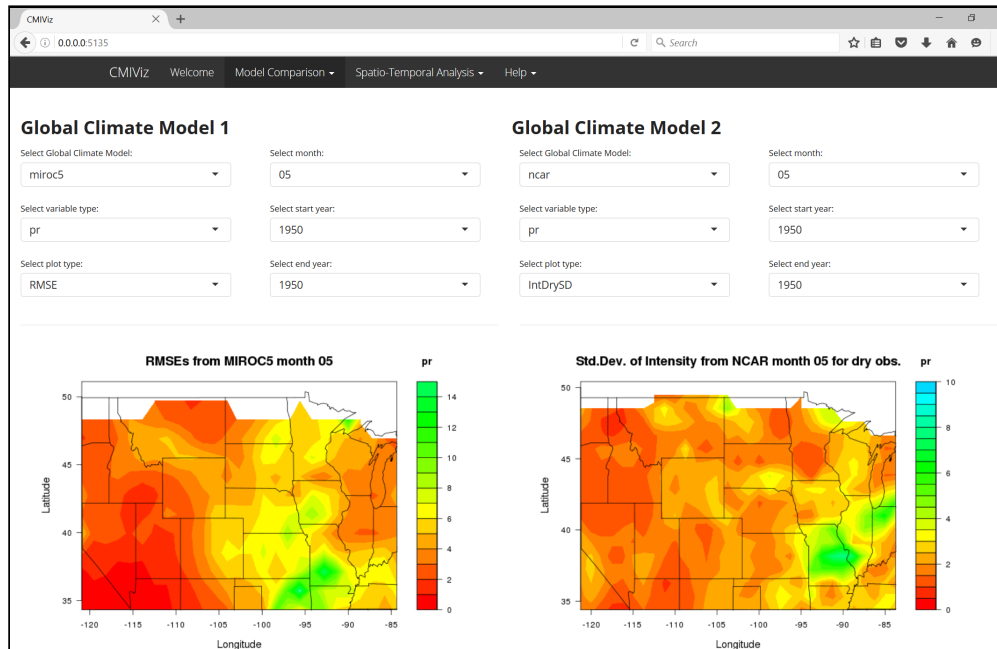


Figure 4.2: CMIP5 Model Comparison in CMIViz.

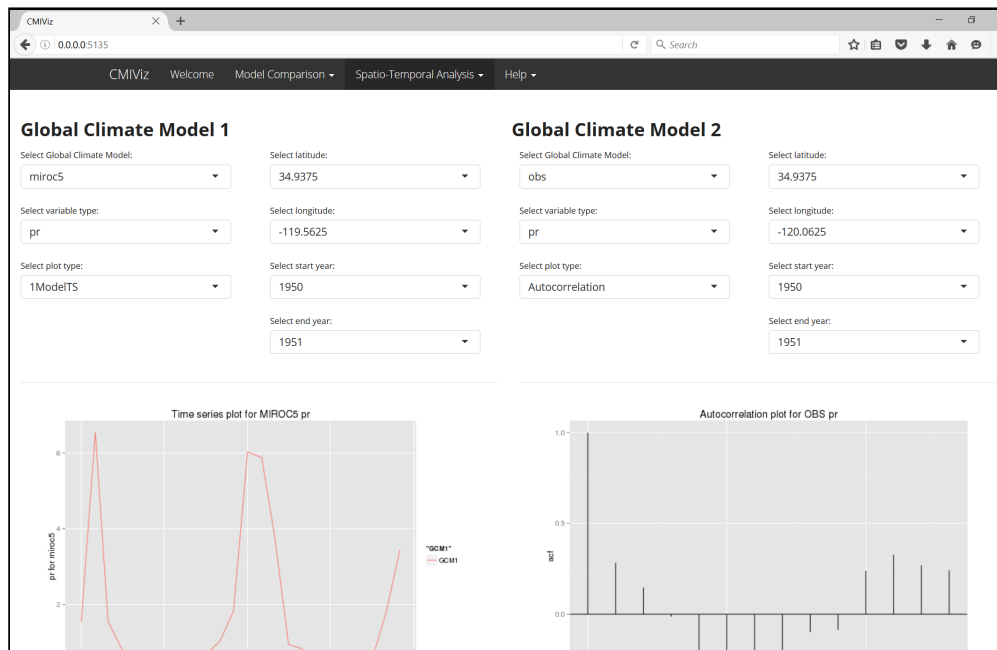


Figure 4.3: Time series analysis in CMIViz.

4.3 Spatial Plots

Basic spatial visualization of climate data on CMIViz is available under ‘Spatio-Temporal Analysis→ Spatial Analysis’, shown in Figure 4.4. As before, the user will be able to select

a climate variable, a GCM, a time point (e.g.: July 2004) and the type of plots. Currently, contour and 3-dimensional surface maps are available. Contour maps give an idea on how the variable of interest is distributed throughout the MRB using a color gradient. A surface map can be seen as a 3-dimensional extension of the contour map where the variable of interest is the third dimension. In addition, the user has an option to plot directional semivariograms [2] of the data at 0° , 45° , 95° , and 135° directions. The data is detrended by performing a linear regression on the latitude and longitude prior to calculating the semivariograms. If the data are spatially correlated, then the observations closer to one another should have a higher correlation than those farther apart. The semivariogram reflects this by calculating the variance of the difference between observations as a function of their euclidean distance.

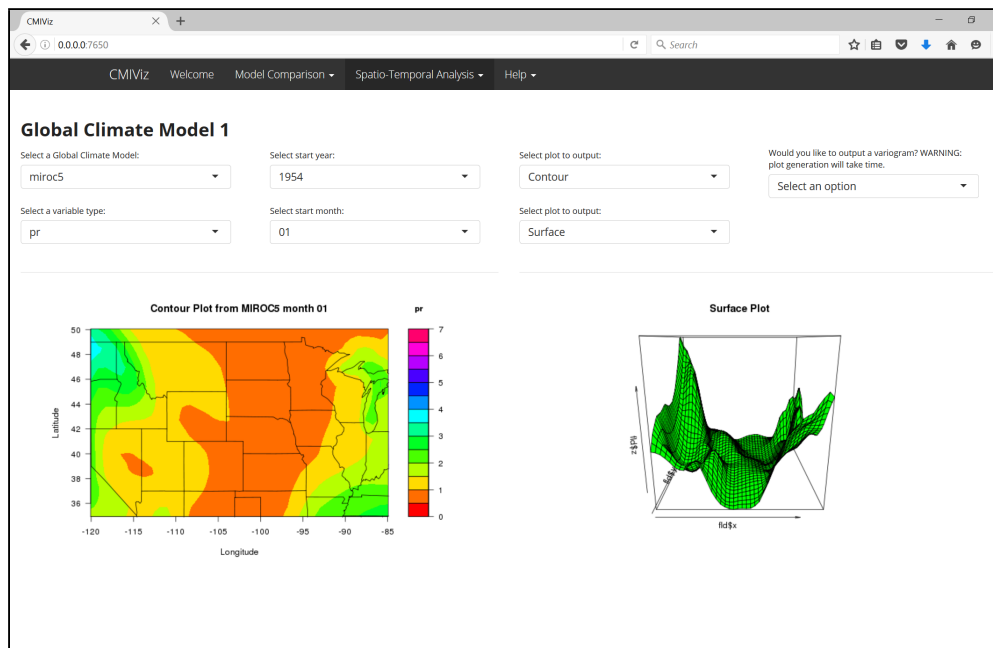


Figure 4.4: Spatial analysis in CMIViz.

4.4 Spatio-Temporal Analysis

Longitude and latitude space-time plots are available on CMIViz to visualize trends in space and time simultaneously. These plots can be accessed on CMIViz at the menu option ‘Spatio-Temporal Analysis→Space-Time Analysis’, as shown in Figure 4.5. The user can subset by model, variable type, a year range, a range of latitude/longitudes, and the type of the plot. If the user selects a latitude space-time plot, the data is spatially averaged over the range of longitudes at each latitude. Figure 4.5 shows latitude and longitude space-time plots of the precipitation data provided by MIROC5 between years 1950 and 2003. While there seems to be less spatial variability across latitudes as we move from north to south, the longitude

space-time plot shows much variability in the precipitation as we move from east to west in MRB.

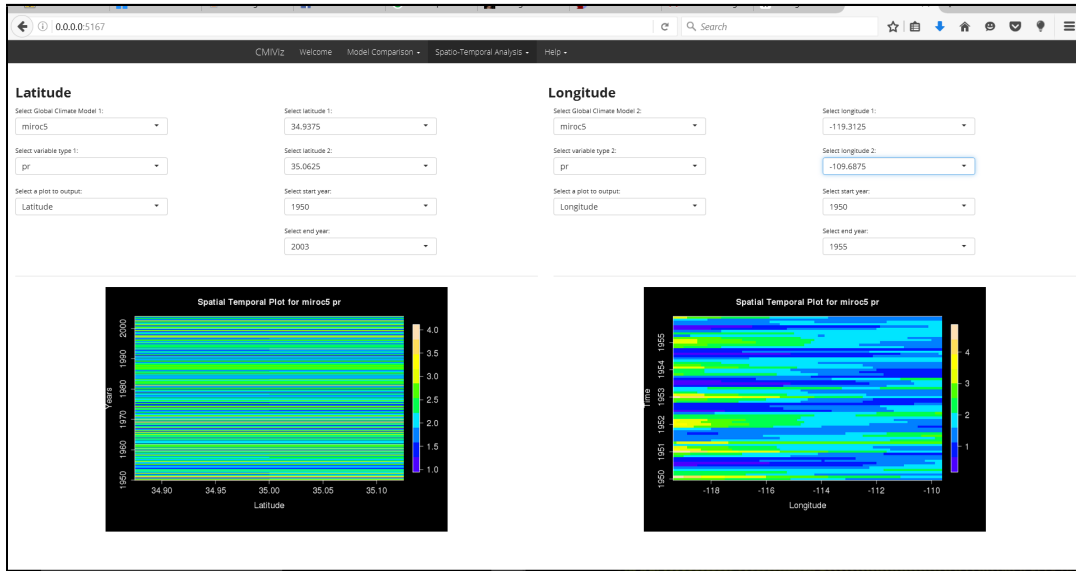


Figure 4.5: Space-time visualization in CMIViz.

4.5 Canonical Correlation Pattern Analysis

Another feature of the spatio-temporal menu is canonical correlation analysis (CCA²; [2]) as shown in Figure 4.6. In CCA, linear combinations of two datasets are considered so that the correlation between the two are maximized. The linear combination with the highest correlation is called the first canonical correlation pattern. Intuitively, CCA determines the similarity between datasets. When calculating CCA in CMIViz, we compare the user selected GCM with the observed dataset. However, because CCA is computationally intensive, we further divide the MRB into three subregions: upper, middle, and lower MRB to reduce the runtime needed for the calculation. In the CCA plot, the first canonical correlation pattern is shown for the model and observed data over time as denoted by different colored lines.

5 Future Directions

CMIViz could be enhanced and improved along several areas in the future. Firstly, since CMIViz is developed using the open source R software and Shiny package, it can be updated as new versions of R and Shiny are available. Also, future enhancements could benefit from additional R packages like ‘leaflet’ that are similar to Shiny. Several additional statistical techniques and visualization options could be added as features to CMIViz. Given the flexible MVC design pattern used in the implementation of CMIViz, adding more features and

²Source code is based on github.com/marchtaylor/sinkr

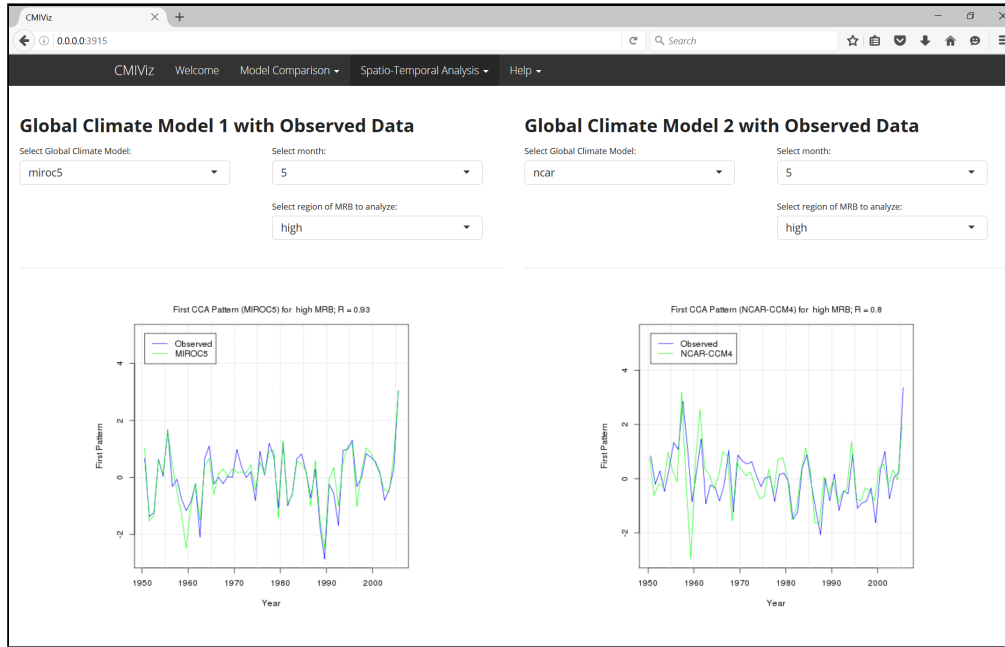


Figure 4.6: CCA in CMIViz

options should be straightforward extensions to the code. Another area worth pursuing is the parallelization of some of the options like calculation of semivariograms in CMIViz. On a less technical note, CMIViz could be made more accessible to a wider audience. As it currently stands, CMIViz can only be run after logging into the UMBC cluster maya. Therefore, users are required to have an account with the UMBC High Performance Computing Facility in order to use CMIViz. Furthermore, the users must create a SOCKS proxy using the browser on their personal computers. This procedure to access CMIViz is not very convenient for a first time user, so an easier way to access could be explored. Future enhancements to CMIViz could also consider such possibilities as the ‘ShinyIO’ website or other online options for hosting CMIViz.

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substantial support from UMBC.

A Complete CMIViz Functionality

CMIViz Model Comparison Plots

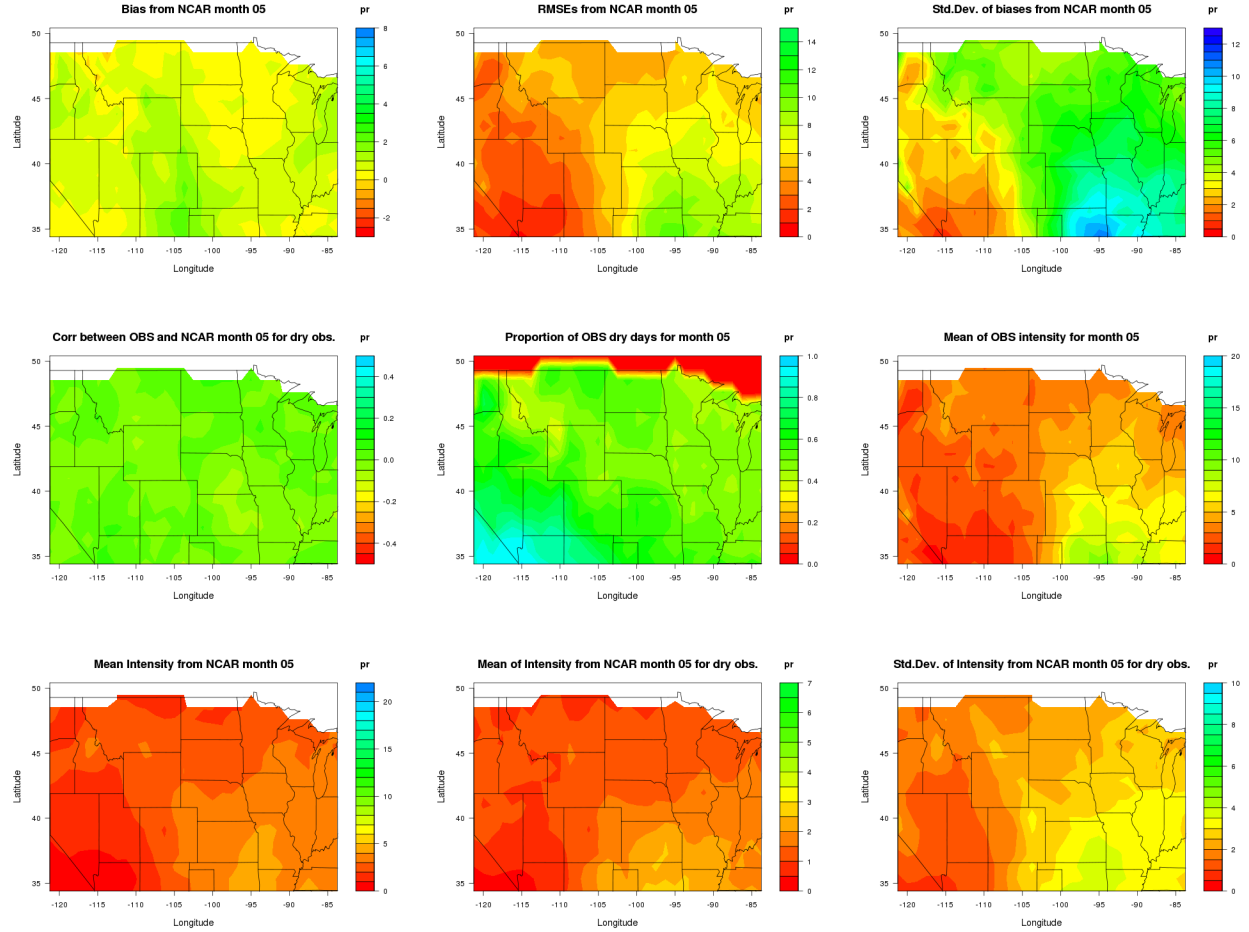


Figure A.1: Calculated comparison statistics are bias, root mean squared error, bias standard deviation, correlation, dry proportion, observed intensity, wet intensity, dry intensity, and dry intensity standard deviation.

CMIViz Temporal Plots

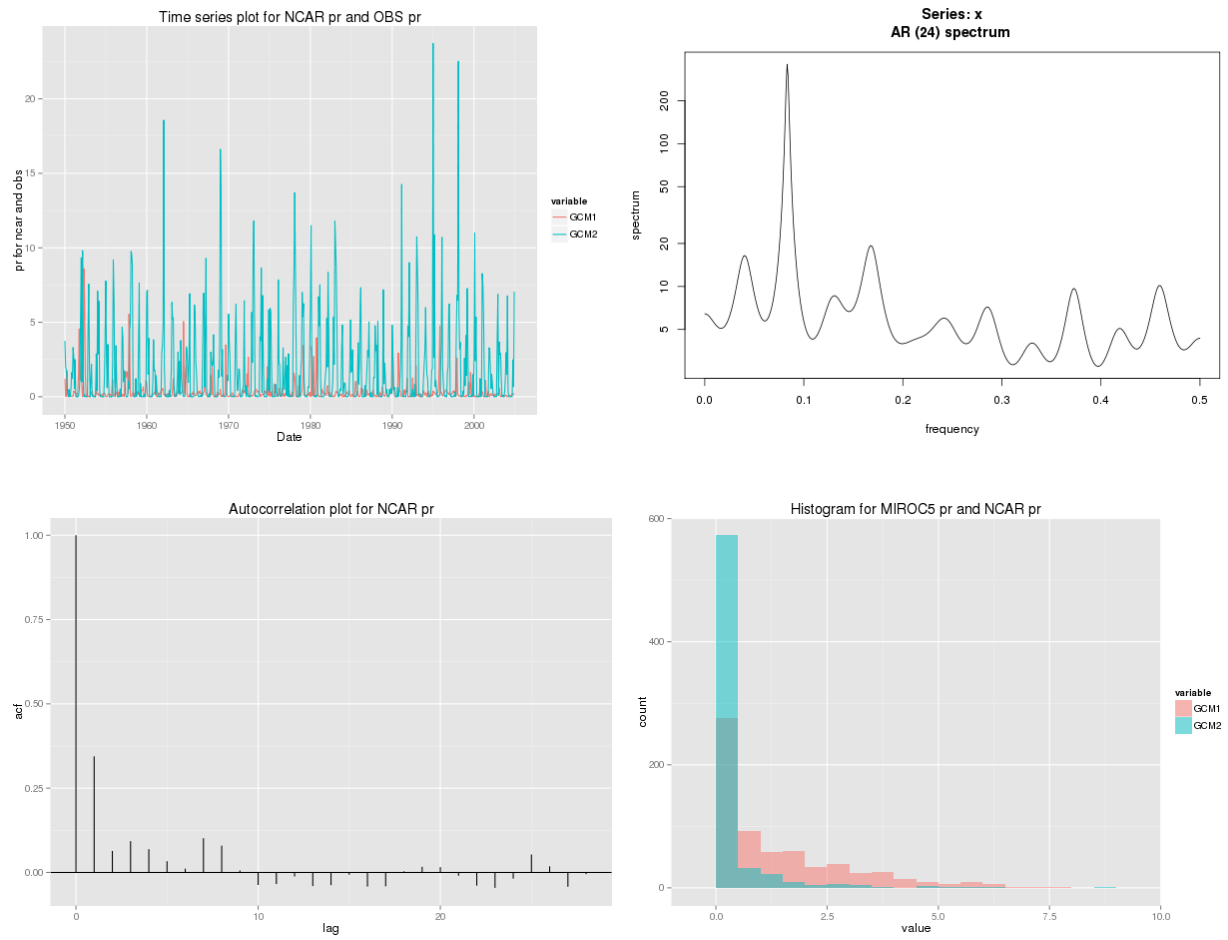


Figure A.2: Plots from this tab allow users to compare models over time.

CMIViz Spatial Maps

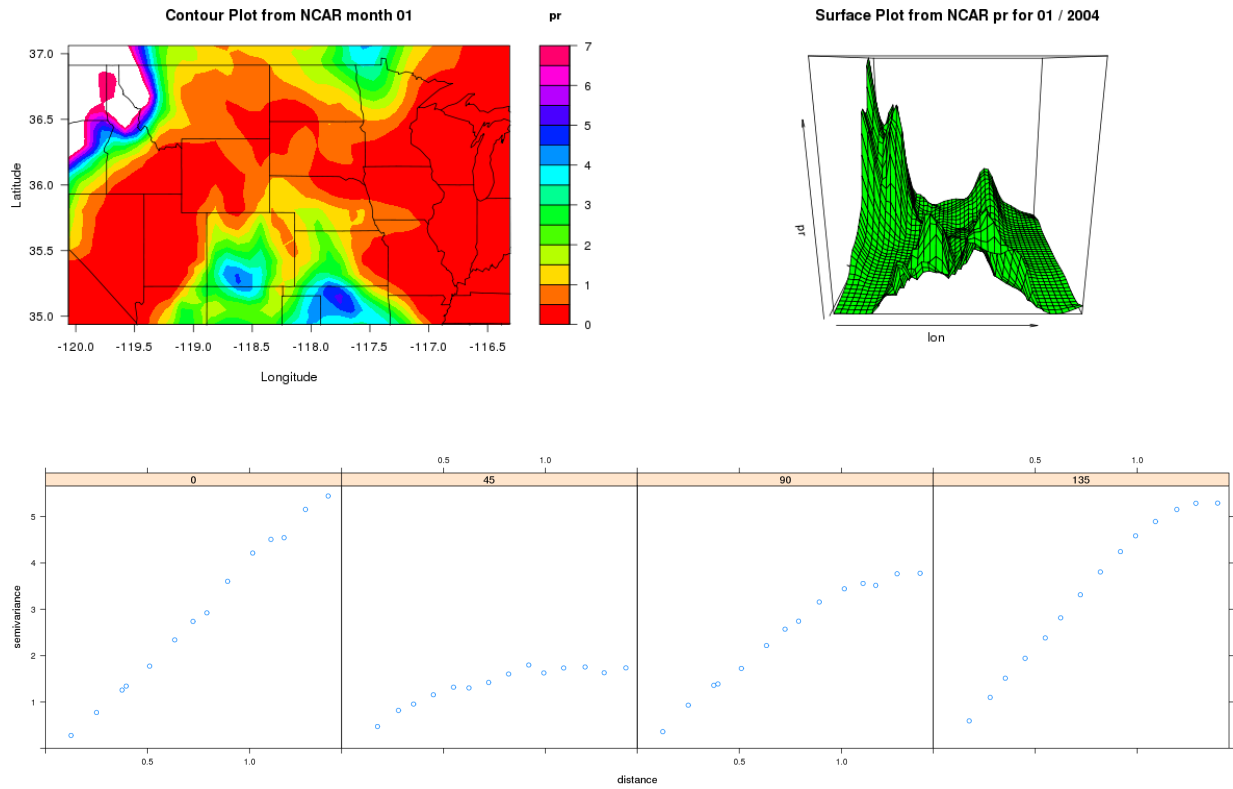


Figure A.3: CMIViz features contour maps, surface plots, and semivariograms.

CMIViz Space-Time Plots

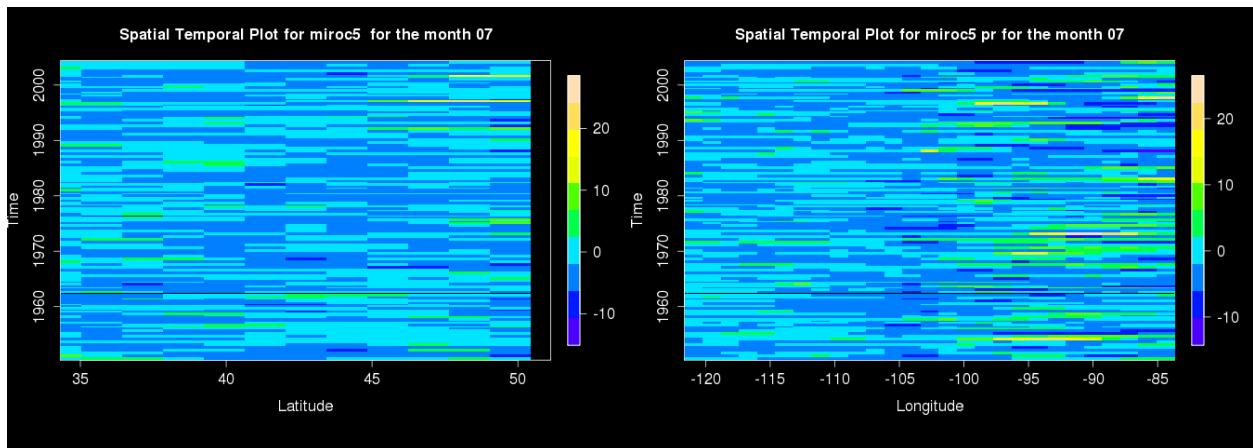


Figure A.4: Space-time plots can average by latitude or longitude based upon user selection.

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